International Journal of Engineering, Management and Humanities (IJEMH)
Volume 6, Issue 2, Mar.-Apr, 2025 pp: 341-343 ISSN: 2584-2145 www.ijemh.com

Filters on Fuzzy Sets

Faleh Abdul Mahdi Jaber
Ministry of Education, Open Educational College, Studies Muthanna Centre, IRAQ

Date of Submission: 20-04-2025 Date of Acceptance: 02-05-2025

ABSTRACT

Fuzzy sets play an important role in various applications of mathematics, especially in the field of artificial intelligence and many other important applications. On the other hand, filters are of great importance in topology and its many applications, but the definition and use of the filter was limited only to classical sets. In this paper we will present a generalization of the concept of filter to fuzzy sets and prove some important properties.

Keywords: Fuzzy set, Filter, Filter-basis

I. INTRODUCTION

Fuzzy logic plays a prominent role in artificial intelligence and other sciences, which has witnessed very important leaps and developments in the recent period. However, making the appropriate decision remains a clear challenge for the algorithms on which the artificial intelligence program works. Filters work in mathematics to find relationships between sets according to strict conditions, which prompted us to define the filter on fuzzy sets, which provides opportunities for broader applications, especially in the field of artificial intelligence.

Fuzzy logic was first created in 1965 when Zadeh[7] from the University of California presented a research paper in which he dealt with the fuzzy set, but it did not receive widespread attention at the time until 1974 came when fuzzy logic was used to organize a steam engine. Then the applications of fuzzy logic continued in many fields[1], the most important of which is intelligence Artificial [3]. With the development of computers and software, there has been an urgent need to create software and systems that can deal with inaccurate information like the human mind does. Hence lies the importance of fuzzy logic, which is the cornerstone of artificial intelligence algorithms[10].

On the other hand, filters play a prominent role in the convergence theory.H. Cartan[6] was the first to know the concept of filters in 1937, and Bourbaki[8]introduced some treatments to it in 1940, but its use in the convergence theory was in 1948 by Choquet[5], which was later called convergence spaces.In the last two decades, many researchers and specialists [2], [4], [9] in the field of mathematics and its applications have presented important research on the applications of filters.

Now, through this research, we have defined the concept of a filter on fuzzy sets and taken some generalizations related to the concept of a filter from regular sets to fuzzy sets, arriving at the concept of a fuzzy neighborhood filter, which we hope will greatly help in the field of artificial intelligence, especially in decision-making algorithms[10-11].

1.1 STRUCTURE

Fuzzy logical: Fuzzy sets or fuzzy logic is the most widely used and widespread concept of traditional sets because they deal with ambiguity, lack of clarity, or actual and definitive belonging.

The fuzzy set gives each element a degree of belonging to a specific characteristic or phenomenon, provided that this degree does not exceed the limits of the interval [12-15].

Definition: let X be a set, $u: X \to [0,1]$ is called the belonging function of the set A and the pair (A, u) is called a fuzzy set on X.We will use the symbol u_A for convenience instead of (A, u).

A fuzzy set A is empty fuzzy set $(\hat{0})$ if and only if $u_A(a) = 0$ for all $a \in X$. Tow fuzzy sets A and B are equal if $u_A(a) = u_B(a)$ for all $a \in X$.A universal fuzzy set A is denoted by the symbol $\tilde{1}$if and only if$u_A(a) = 0$ for all $a \in X$.

Now we mention the most important operations on fuzzy sets.For this reason, suppose we have two fuzzy sets A and B on the universal set X, then;

i. A fuzzy set A subset of a fuzzy set B or $A \subseteq B$ if and only if $u_A(a) \le u_B(a)$ for all $a \in X$.
ii. The complement of the fuzzy set A is denoted by the symbol A^c where$A^c$where m is defined as follows:$u_{A^c}(a) = 1 - u_A(a)$ for all $a \in X$.
iii. The intersection of two fuzzy sets A and B on the universal set X is defined as follows:$u_{A\cap B}(a) = \min\{u_A(a), u_B(a)\}$ for all $a \in X$.
iv. While the union is defined between of two fuzzy sets A and B on the universal set X is defined

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as follows: $u_{A\cup B}(a) = \max\{u_A(a), u_B(a)\}$ for all $a \in X$.

2.2 The Filter:

A filter in A is a non-empty set $\mathcal{F}$ of subsets of A, such that:

1. The empty set is not belong to $\mathcal{F}$
2. $F_1 \in \mathcal{F}$ and $F_1 \subseteq F_2 \to F_2 \in \mathcal{F}$
3. $F_1, F_2 \in \mathcal{F} \to F_1 \cap F_2 \in \mathcal{F}$.

A filter-basis in a space A is a nonempty set β of subsets of A that satisfies the conditions: $\emptyset \notin \beta$, and for all $\mathcal{B}_1, \mathcal{B}_2 \in \beta$ there exists $\mathcal{B}_3 \in \beta$ such that $\mathcal{B}_1 \cap \mathcal{B}_2 \supseteq \mathcal{B}_3$.

3. Fuzzy Filter

From now on, let's agree on the following symbols: We will denote the family of all fuzzy sets on set X by I^X

2.2 Definition

Let X be a universal set under study, a fuzzy filter on X is a family $\delta = \{A \in I^X\}$ which meets the following three conditions:

1. The empty fuzzy set is not belong to δ
2. If $A \in \delta$ and $A \subset B$ then $B \in \delta$
3. If $A, B \in \delta$, then $A \cap B \in \delta$.

2.2 Example

Let $X = \{x_1, x_2, x_3\}$ and

$$A = \{< x_1, 0.8 >, < x_2, 0.9 >, < x_3, 0.7 >\}$$
$$B = \{< x_1, 0.5 >, < x_2, 0.9 >, < x_3, 1 >\}$$
$$C = \{< x_1, 0.8 >, < x_2, 0.2 >, < x_3, 0.6 >\}$$
$$D = \{< x_1, 0.6 >, < x_2, 0.5 >, < x_3, 0.4 >\}$$
$$E = \{< x_1, 0.3 >, < x_2, 0.3 >, < x_3, 0.2 >\}$$

If $\delta = \{A, D, E\}$

Then δ is a fuzzy filter since;

1. The empty fuzzy set is not belong to δ
2. $C \in \delta$ and $C \subset A$ then $A \in \delta$, $C \in \delta$ and $C \subset B$ then $B \in \delta$, $B \in \delta$ and $B \subset A$ then $A \in \delta$.
3. $A \cap B = B \in \delta$, $A \cap C = C \in \delta$, $B \cap C = C \in \delta$.

2.2 Example

Suppose we have a non-empty set X and a family δ of fuzzy sets defined on X as follows:

$$\delta = \{A \in I^X : u_A(x) \geq \tfrac{1}{2} \; for \; all \; x \in X\}$$

It is clear that δ satisfies the three conditions of definition 5 and hence δ is a fuzzy filter.

Since every filter has a basis, the fuzzy filter also has a basis, it is now appropriate to define the basis of the fuzzy filter as follows:

2.2 Definition

The family β of a fuzzy sets on X is said to be fuzzy filter-basis if the following two conditions are met;

1. The empty fuzzy set is not belong to β
2. For all $A_1, A_2 \in \beta$ there exists $A_3 \in \beta$ such that $A_1 \cap A_2 \supseteq A_3$.

2.2 Definition

Let δ_1, δ_2 are two fuzzy filters in a set X, we say that δ_1 is finer than δ_2 or δ_2 is coarser than δ_1 if $\delta_1 \supseteq \delta_2$.

2.2 Definition

Let δ_1, δ_2 are two fuzzy filters on a set X . If $\delta_1 \cap \delta_2 \neq \emptyset$ then the supermum of the two fuzzy filters is $\delta_1 \vee \delta_2 = \{u_1 \cap u_2 : u_1 \in \delta_1, u_2 \in \delta_2\}$.

2.2 Definition

Let δ_1, δ_2 are two fuzzy filters on a sets X, Y respectively, then we define the direct product $\delta_1 \times \delta_2$ is the fuzzy filter on $X \times Y$ generated by; $\{u_1 \times u_2 : u_1 \in \delta_1, u_2 \in \delta_2\}$.

Now we move to the definition of a special type of fuzzy filter, which is the neighborhood filter, but before this we must mention the definition of fuzzy topology;

2.3 Definition

Let X be a set and $\mathcal{T} \in [0,1]^X$ be a family of fuzzy sets, then $\mathcal{T}$ is called fuzzy topology if the following three conditions are met;

1. $\tilde{0} \in \mathcal{T}, \tilde{1} \in \mathcal{T}$
2. For all $A, B \in \mathcal{T}$ $then$ $A \cap B \in \mathcal{T}$
3. For all $A_i \in \mathcal{T}$ $then$ $\bigcup_i (A_i) \in \mathcal{T}$ $where$ $i = 1,2, \ldots$

The pair $(X, \mathcal{T})$ is a fuzzy topological space.

Similar to the definition of the neighborhood of a point in topology defined on a classical set, we will define the neighbors of a fuzzy point.

2.2 Definition

Let $(X, \mathcal{T})$ be a fuzzy topological space, the fuzzy point $\mathcal{P}_x^\alpha$ is a fuzzy set defined as follows;

$$\mathcal{P}(y) = \begin{cases} \alpha & if \; y = x \\ 0 & if \; y \neq x \end{cases}, \qquad \text{where } 0 < \alpha \leq 1$$

And so that we can write each fuzzy set A on X as follows;

$$A = \bigcup \{\mathcal{P}_x^\alpha : 0 < \alpha \leq u_A(x)\}$$

2.2 Definition

Let $(X, \mathcal{T})$ be a fuzzy topological space. A fuzzy set A is called a neighborhood of a fuzzy point $\mathcal{P}_x^\alpha$ in $(X, \mathcal{T})$ if and only if there exists a fuzzy set $\mathcal{W} \in \mathcal{T}$ such that:

$$\mathcal{P}_x^\alpha \in \mathcal{W} \subseteq A$$

We will denote the family of all neighbors of the fuzzy point $\mathcal{P}_x^\alpha$ with the symbol $\mathcal{N}(\mathcal{P}_x^\alpha)$

3.9 Proposition

Let $(X, \mathcal{T})$ be a fuzzy topological space and $\mathcal{N}(\mathcal{P}_x^\alpha)$ the collection of all $\mathcal{T}-$neighborhood of a fuzzy point $\mathcal{P}_x^\alpha$. Then $\mathcal{N}(\mathcal{P}_x^\alpha)$ is a fuzzy filter on X.

Proof:

1. Since $0 < \alpha \le 1$, then $\tilde{0} \notin \mathcal{N}(\mathcal{P}_x^\alpha)$
2. Let $A \in \mathcal{N}(\mathcal{P}_x^\alpha)$ and $A \subseteq B$,

Since $A \in \mathcal{N}(\mathcal{P}_x^\alpha)$, then there exists a fuzzy set $\mathcal{W} \in \mathcal{T}$ such that

$$\mathcal{P}_x^\alpha \in \mathcal{W} \subseteq A$$

$$\mathcal{P}_x^\alpha \in \mathcal{W} \Longrightarrow \alpha \le u_w(x)$$

$$\mathcal{W} \subseteq A \Longrightarrow u_w(x) \le u_A(x)$$

But $A \subseteq B \Longrightarrow u_A(x) \le u_B(x)$

This we have $\mathcal{P}_x^\alpha \in \mathcal{W} \subseteq B$ and therefor $A \in \mathcal{N}(\mathcal{P}_x^\alpha)$

1. Let $A, B \in \mathcal{N}(\mathcal{P}_x^\alpha)$ and we have to prove that $A \cap B \in \mathcal{N}(\mathcal{P}_x^\alpha)$

From $A \in \mathcal{N}(\mathcal{P}_x^\alpha)$, then there exists a fuzzy set $\mathcal{W}_1 \in \mathcal{T}$ such that

$$\mathcal{P}_x^\alpha \in \mathcal{W}_1 \subseteq A$$

And from $B \in \mathcal{N}(\mathcal{P}_x^\alpha)$, then there exists a fuzzy set $\mathcal{W}_2 \in \mathcal{T}$ such that

$$\mathcal{P}_x^\alpha \in \mathcal{W}_2 \subseteq B$$

$$\mathcal{P}_x^\alpha \in \mathcal{W}_1 \; and \; \mathcal{P}_x^\alpha \in \mathcal{W}_2 \Longrightarrow \mathcal{W}_1 \cap \mathcal{W}_2 \neq \emptyset$$

Since $\mathcal{W}_1 \subseteq A \Longrightarrow u_{w_1}(x) \le u_A(x)$

Since $\mathcal{W}_2 \subseteq B \Longrightarrow u_{w_2}(x) \le u_B(x)$

But $\min\{u_{w_1}(x), u_{w_2}(x)\} \le \min\{u_A(x), u_B(x)\}$

This we have $\mathcal{W}_1 \cap \mathcal{W}_2 \le A \cap B$

Let $\mathcal{W} = \mathcal{W}_1 \cap \mathcal{W}_2$

Now we have it$\mathcal{P}_x^\alpha \in \mathcal{W} \le A \cap B$

That means$A \cap B \in \mathcal{N}(\mathcal{P}_x^\alpha)$

The found fuzzy neighborhood filter can be used in approximation theory in the future.

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