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Filters on Fuzzy Neutrosophic Soft Sets and Using in Artificial Intelligence

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ABSTRACT: In this research, we present a definition of a filter, not in classical sets, but in neutrosophic sets. We will then present some examples and important properties related to the topic of filters. For further generalizations applicable to artificial intelligence, we will apply these filters to neutrosophic soft set, followed by a discussion of decision-making algorithms.

KEYWORDS: Soft set, NeutrosophicSets ,Neutrosophic Soft Sets, Filter, Filter-basis.

I. INTRODUCTION

Until recently, all mathematical algebraic structures were built upon ordinary classical sets, in which any element has only two options: either it belongs to or it does not belong to that set under study. However, in 1965, Zadeh[ ]introduced the idea of fuzzy logic, which assigns each element a degree of belonging to the group, and which was then called the fuzzy set. It is natural that most of the algebraic structures that were known from ordinary sets would be generalized to chaotic sets, which have found wide applications in other sciences, especially in the field of artificial intelligence.Atanassovhad a different opinion, which was the necessity of giving each element a degree of belonging and a degree of non-belonging to the set, and he became known as intuitive groups, which were a generalization of Intuitionistic fuzzy set. However, there are many phenomena or applications where it is difficult to determine whether an element belongs or does not belong, or where there is ambiguity in determining its belonging to something specific. This led Smarandache to generalize the intuitive sets, which included two functions, to the neutrosophic sets, which include a third function in addition to belonging and non-belonging, namely the neutrality or uncertainty function. In 1999, Molodtsov introduced the concept of soft sets to deal with complex problems that require many parameters, or in short, to deal with data that does not seem entirely clear.

On the other hand, filters play a prominent role in the convergence theory. H. Cartan[6] was the first to know the concept of filters in 1937,

andBourbaki[8]introduced some treatments to it in 1940, but its use in the convergence theory was in 1948 by Choquet[5], which was later called convergence spaces.In the last two decades, many researchers and specialists [2], [4], [9] in the field of mathematics and its applications have presented important research on the applications of filters.

In this research, we will introduce the concept of filtering on neutrosophy groups and some of its basic properties, which is expected to greatly improve decision-making algorithms in artificial intelligence.

II. STRUCTURE

The reader is expected to have a complete understanding of the fuzzy sets that Zadeh defined, which give each element a specific degree of belonging confined within the period [0,1]. For those who want more information about fuzzy logic, we recommend reviewing sources 5 and 7.

Therefore, we move directly to the definition of neutral sets, which are a generalization of fuzzy sets and intuitive sets. Many issues in engineering, medicine, sociology, and other fields involve varying degrees of uncertainty or neutrality. Neutrosophic sets address these issues, assigning three degrees to eachelement, each belonging to a closed interval [0,1]. These values represent the degree of belonging $T(x)$, the degree of neutrality or uncertainty $I(x)$, and the degree of non-belonging $F(x)$.

2.1 Definition

A fuzzy neutrosophic set A on the universe set X is defined as:

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$A = \{< x, (\mathcal{T}_A(x), I_A(x), \mathcal{F}_A(x)) >: x \in X\}$, *where* $\mathcal{T}, \mathcal{I}, \mathcal{F}: X \longrightarrow [0,1]$ *and* $0 \le \mathcal{T}_A(x) + I_A(x) + \mathcal{F}_A(x) \le 3$.

Let's now agree on some symbols for ease of use;

$$\max\{\mathcal{T}_A(x), \mathcal{T}_B(x)\} = \mathcal{T}_{\text{mx}}(x)$$
$$\max\{I_A(x), I_B(x)\} = I_{\text{mx}}(x)$$
$$\max\{\mathcal{F}_A(x), \mathcal{F}_B(x)\} = \mathcal{F}_{\text{mx}}(x)$$
$$\min\{\mathcal{T}_A(x), \mathcal{T}_B(x)\} = \mathcal{T}_{\text{mn}}(x)$$
$$\min\{I_A(x), I_B(x)\} = I_{\text{mn}}(x)$$
$$\min\{\mathcal{F}_A(x), \mathcal{F}_B(x)\} = \mathcal{F}_{\text{mn}}(x)$$

2.2 Definition

Let $A = \{< x, (\mathcal{T}_A(x), I_A(x), \mathcal{F}_A(x)) >: x \in X\}$ and $B = \{< x, (\mathcal{T}_B(x), I_B(x), \mathcal{F}_B(x)) >: x \in X\}$ are two neutrosophic sets on X then;

1. $A \subseteq B$ if for all $x \in X$ we have $\mathcal{T}_A(x) \le \mathcal{T}_B(x)$, $I_A(x) \ge I_B(x)$ and $\mathcal{F}_A(x) \ge \mathcal{F}_B(x)$
2. $A \sqcup B = \{< x, (\mathcal{T}_{mx}(x), I_{mn}(x), \mathcal{F}_{mn}(x)) > : x \in X\}$
3. $A \sqcap B = \{< x, (\mathcal{T}_{mx}(x), I_{mn}(x), \mathcal{F}_{mn}(x)) > : x \in X\}$
4. $A^c = \{< x, (\mathcal{F}_A(x), 1 - I_A(x), \mathcal{T}_A(x)) >: x \in X\}$
5. A is said to be universe fuzzy neutrosophic set and denoted this by 1_A , If $\forall\, x \in X$, we have $\mathcal{T}_A(x) = 1$, $I_A(x) = 0$ and $\mathcal{F}_A(x) = 0$
6. A is said to be empty fuzzy neutrosophic set and denoted this by 0_A , If $\forall\, x \in X$, we have $\mathcal{T}_A(x) = 0$, $I_A(x) = 1$ and $\mathcal{F}_A(x) = 1$

2.3 Definition

Let X be a universe set and E be a set of parameters. A pair (F, E) is called a soft set over X if and only if F is a function from E into the set of all subsets of X , i.e. $: E \to P(X)$, where $P(X)$ is the power set of X .

2.4 Definition

Let X be the universe set and E a set of parameters. Consider a non-empty set $A, A \subset E$. If $FNs(X)$ denote the set of all fuzzy neutrosophic sets of , the collection $(f.E)$ is termed to be the fuzzy neutrosophic soft set over X where $f: A \to FNs(X)$.

This concept has been modified by Deli and Broumi [18] as given below .

2.5 Definition

Let $FN(X)$ refer to all neutrosophic sets defined on the set under study Xand let us have the set E whose elements are parameters (things, attributes, etc.).The neutrosophicsoft sets (f, E) are a function of domain E or any subset thereof, and its codomain is $FN(X)$.

However, Daly had a clearer definition of neutrosophicsoft sets, which we will denote by the symbol (X), which states:

2.6 Definition

Let X be an universe set and E a set of parameters. Then a fuzzy neutrosophic soft set H over X is a set defined by a set valued function $f_H$representing a mapping $f_H : E \to FNS(X)$,where f_H iscalled approximate function of the neutrosophic soft set H. In other words, the neutrosophic soft set is a parametrized family of some elements of the set $FNs(X)$and therefore it can be written as a set of ordered pairs,

$H = \{(e, \{< x, \mathcal{T}_{f_{H(e)}}(x), I_{f_{H(e)}}(x), \mathcal{F}_{f_{H(e)}}(x) > : x \in X\}) : e \in E\}$where

$\mathcal{T}_{f_{H(e)}}(x), I_{f_{H(e)}}(x), \mathcal{F}_{f_{H(e)}}(x) \in [0,1]$, respectively called the truth-membership ,indeterminacy-membership, falsity-membership function of $f_H(e)$, and $0 \le \mathcal{T}_{f_{H(e)}}(x) + I_{f_{H(e)}}(x) + \mathcal{F}_{f_{H(e)}}(x) \le 3$, $\forall e \in E$.

2.7 Example

Let $X = \{a, b, c\}$ be a universe set and $E = \{e_1, e_2\}$ and let $f: E \to FNs(X)$

$$H = \{(e, \{< x, \mathcal{T}_{f_{H(e)}}(x), I_{f_{H(e)}}(x), \mathcal{F}_{f_{H(e)}}(x) >: x \in X\})\}$$

$$f_{H(e_1)} = \{< a, (0.3,0.4,0.7) >, < b, (0.5,0.6,0.4) >, < c, (0.6,0.5,0.4) >\}$$

$$f_{H(e_2)} = \{< a, (0.6,0.1,0.5) >, < b, (0.5,0.3,0.9) >, < c, (0.8,0.5,0.3) >\}$$

then $H = \{[e_1, f_{H(e_1)}], [e_2, f_{H(e_2)}]\}$is an $FNSs$.

2.8 Definition

A filter in A is a non-empty set Γ of subsets of A, such that:

1. Γdoes not contain the empty set.
2. If $N_1 \subseteq N_2$ *and* $N_1 \in \Gamma$ then $N_2 \in \Gamma$
3. For all $N_1, N_2 \in \Gamma$ *then* $N_1 \cap N_2 \in \Gamma$.

A filter-basis in a space A is a nonempty set β of subsets of A that satisfies the conditions:$\emptyset \notin \beta$,and for all $\mathcal{B}_1, \mathcal{B}_2 \in \beta$ there exists $\mathcal{B}_3 \in \beta$ such that $\mathcal{B}_1 \cap \mathcal{B}_2 \supseteq \mathcal{B}_3$.

III. Fuzzy Neutrosophic Filter

We now offer a generalization of the filter from classic sets to fuzzy neutrosophic soft sets. From now on, let's agree that the symbol $FN(U)$

refers to the family of all fuzzyneutrosophic sets on the set U, while the symbol $FNS(U)$ refers to the family of all fuzzy neutrosophic soft sets on the global set U and the set of parameters under study E.

3.1 Definition

Let U be a universal set under study, a fuzzy neutrosophic filter on U is a family $Ⅎ = \{B \in FN(U)\}$, this family of fuzzy neutrosophic sets achieves the following three things:

1. $0_B \notin Ⅎ$
2. If $B_1 \in Ⅎ$ and $B_1 \subset B_2$ then $B_2 \in Ⅎ$
3. If $B_1, B_2 \in Ⅎ$, then $B_1 \cap B_2 \in Ⅎ$.

3.2 Example

Let $U = \{a, b, c\}$, and

$B_1 = \{< a, (0.3,0.5,0.6) >, < b, (0.1,0.5,0.9) >, < c, (0.8,0.5,0.7) >\}$

$B_2 = \{< a, (0.5,0.4,0.5) >, < b, (0.4,0.3,0.8) >, < c, (0.9,0.4,0.3) >\}$

$B_3 = \{< a, (0.6,0.5,0.5) >, < b, (0.3,0.7,0.2) >, < c, (0.6,0.5,0.9) >\}$

$B_4 = \{< a, (0.8,0.5,0.7) >, < b, (0.5,0.3,0.9) >, < c, (0.8,0.5,0.1) >\}$

$B_5 = \{< a, (0.6,0.4,0.3) >, < b, (0.4,0.3,0.7) >, < c, (0.9,0.1,0.3) >\}$

$B_6 = \{< a, (0.9,0.5,0.5) >, < b, (0.3,0.7,0.2) >, < c, (0.6,0.5,0.9) >\}$

$B_7 = \{< a, (1,0,0) >, < b, (1,0,0) >, < c, (1,0,0) >\}$

Let's assume we have the following assemblages of those neutrosophic sets;

$Ⅎ_1 = \{B_1, B_3, B_4, B_5\}$, $Ⅎ_2 = \{B_1, B_2, B_5, B_7\}$, $Ⅎ_3 = \{B_5, B_7\}$

Now let's check the above neutrosophic families to see if any of them represent a a fuzzy neutrosophic filter on a set U under study,

It is clear that $Ⅎ_1$ does not satisfy the second condition of the definition of a fuzzy filter because $B_1 \in Ⅎ_1$ and $B_1 \subset B_2$ *but* $B_2 \notin Ⅎ_1$, therefore, $Ⅎ_1$ does not represent a fuzzy neutrosophic filter on a set U.

We note that both $Ⅎ_2$ and $Ⅎ_3$ represent a fuzzy neutrosophic filter on U due to the availability of the three conditions mentioned in definition (2.2).

3.3 Proposition

Let X be a non-empty set and $\alpha_1 \in (0,1], \alpha_2, \alpha_3 \in [0,1]$, then the family ψ of fuzzyneutrosophic sets on the set X that

$\psi = \{A \in FN(U): \mathcal{T}_A(x) \geq \alpha_1, I_A(x) \leq \alpha_2, \mathcal{F}_A(x) \leq \alpha_3 \ \forall \ x \in X\}$. it represents a fuzzy neutrosophic filter on X.

Proof:

1. Since $\alpha_1 \in (0,1]$ then $0 < \alpha \leq 1$, and therefor $\tilde{0} \notin \psi$
2. Let $A \in \psi$ and $A \subseteq B$,

Since $A \in \psi$, then there exists $\alpha_1 \in (0,1], \alpha_2, \alpha_3 \in [0,1]$ such that

$\mathcal{T}_A(x) \geq \alpha_1, I_A(x) \leq \alpha_2, \mathcal{F}_A(x) \leq \alpha_3 \ \forall \ x \in X$

But $A \subseteq B$, this for all $x \in X$ we have;

$\mathcal{T}_A(x) \leq \mathcal{T}_B(x) \geq \alpha_1$,

$I_A(x) \geq I_B(x) \leq \alpha_2$,

$\mathcal{F}_A(x) \geq \mathcal{F}_B(x) \leq \alpha_3$

And so that $B \in \psi$

1. Let $A, B \in \psi$ and we have to prove that $A \cap B \in \psi$

From $A \in \psi$, then there exists $\alpha_1 \in (0,1], \alpha_2, \alpha_3 \in [0,1]$ such that

$T_A(x) \geq \alpha_1, I_A(x) \leq \alpha_2, F_A(x) \leq \alpha_3 , \forall \ x \in X$

And from $B \in \psi$, then there exists $\gamma_1 \in (0,1], \gamma_2, \gamma_3 \in [0,1]$ such that

$T_B(x) \geq \gamma_1, I_B(x) \leq \gamma_2, F_B(x) \leq \gamma_3 , \forall \ x \in X$

Let $\delta_1 = \min\{\alpha_1, \gamma_1\}$, then $\delta_1 \in (0,1]$,

$\delta_2 = \max\{\alpha_2, \gamma_2\}$, then $\delta_2 \in [0,1]$

And $\delta_3 = \max\{\alpha_3, \gamma_3\}$, then $\delta_3 \in [0,1]$

This we have $\min\{T_A(x), T_B(X)\} \geq \delta_1$

$\max\{I_A(x), I_B(x)\} \leq \delta_2$

$\max\{F_A(x), F_B(x)\} \leq \delta_3$

And therefor that $A \cap B \in \psi$

So that it represents a fuzzy neutrosophic filter on X.

Now we will move on to defining the basis of the fuzzy neutrosophic filter accordingof the fundamentals of defining filters on standard sets. Since every filter has a basis, the fuzzy neutrosophic filter also has a basis, it is now appropriate to define the basis of the fuzzy neutrosophic filter as follows:

3.4 Definition

The family β of a fuzzy neutrosophic sets on U is said to be fuzzy neutrosophic filter -basis if the following two conditions are met;

1. $0_B \notin \beta$
2. For all $A_1, A_2 \in \beta$ there exists $A_3 \in \beta$ such that $A_1 \cap A_2 \supseteq A_3$.

It is clear that for every fuzzy neutrosophic filter there is at least one base that generates that filter.

3.5 Definition
Let ψ_1 ,ψ_2 are two fuzzy neutrosophic filters in a set U, we say that ψ_1 is finer than ψ_2 or ψ_2 is coarser than ψ_1 if $\psi_1 \supseteq \psi_2$.
Referring to the previous example (2.3), filter $Ⅎ_2$ is finer than filter $Ⅎ_3$, or filter $Ⅎ_3$ is coarser than filter $Ⅎ_2$.

3.6 Definition
Let ψ_1 ,ψ_2 are two fuzzy neutrosophic filters on a set U . If $\psi_1 \cap \psi_2 \neq \emptyset$ then the supermum of the two fuzzy neutrosophic filters is $\psi_1 \vee \psi_2 = \{A_1 \cap A_2 : A_1 \in \psi_1, A_2 \in \psi_2\}$.

3.7 Definition
Let ψ_1 ,ψ_2 are two fuzzy neutrosophic filters on a sets X, Y respectively, then we define the direct product $\psi_1 \times \psi_2$ is the fuzzy filter on $X \times Y$ generated by;

$$\{A_1 \times A_2 : A_1 \in \psi_1, A_2 \in \psi_2\}.$$

Having generalized the basic and important concepts of filters from regular sets to neutrosophic sets, we now move on to a more general definition of filters on soft sets, specifically fuzzy neutrosophicsoft sets, given the crucial importance of these sets in artificial intelligence topics.

3.8 Definition
Let X be an universe set and E a set of parameters and let H be a fuzzy neutrosophic soft set over X . Then the family of fuzzy neutrosophic soft sets$Ⅎ \subseteq FNs(X)$ is said to be a fuzzy neutrosophic soft filter on U if $f_H(e)$ is a fuzzy neutrosophicfilter $\forall e \in E$.

IV. Applications

The decision tree algorithm is an important and common algorithm that relies on data collection. The roots of the tree in the algorithm are the nodes, and the attributes represent the nodes. Each branch in the tree represents a decision path, with the decision at the end of that branch. However, a challenge lies in selecting the best option from among the available attributes to create the optimal decision branch. In other words, the better the chosen model, the better and less random the decision-making process. These characteristics will be the parameters we discussed earlier. If the input information can form a fuzzy neutrosophic soft filter, then this branch is ideal for decision-making. If there is more thanone branch, the finer filter can maximize the benefit of making the appropriate decision, avoiding the entropy method, which has many problems.

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